LEBANESE AMERICAN UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS

MTH 201 - CALCULUS III

Exam II, Spring 2015

Duration: 60 minutes

INSTRUCTIONS: This exam consists of 8 pages and 7 problems. Check that none is missing. Answer the questions in the space provided for each problem; if more space is needed, you may use the back pages. To receive full credits, you have to justify your answers.

Student's Name: ____________

Student's ID: ____________

Calculator

Exam II

Grading scheme
(Keep it empty)

Question 1	/12
Question 2	/13
Question 3	/15
Question 4	/20
Question 5	/15
Question 6	/20
Question 7	/5
Total	/100

1. **[12 Points]** Show that the series $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$ converges conditionally.

* $\cdot$ $\displaystyle\lim_{n\to+\infty}\sqrt{n+1}-\sqrt{n} = \lim_{n\to+\infty}\left(\sqrt{n+1}-\sqrt{n}\right)\left(\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}\right)$

$\displaystyle = \lim_{n\to+\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}} = 0$ 2

2 $\displaystyle\left(\sqrt{n+1}-\sqrt{n}\right)' = \frac{1}{2\sqrt{n+1}} - \frac{1}{2\sqrt{n}} = \frac{2\overbrace{(\sqrt{n}-\sqrt{n+1})}^{-ve}}{2\sqrt{n+1}\cdot 2\sqrt{n}} < 0$

2 $\Rightarrow \sqrt{n+1}-\sqrt{n}$ is $\downarrow$

$\sqrt{n+1}-\sqrt{n} > 0$

The series is alt.

$\cdot$ By Leibniz's Test, it converges. 2

* $\displaystyle\sum_{n=1}^{+\infty}\left|(-1)^n\cdot\left[\sqrt{n+1}-\sqrt{n}\right]\right| = \sum_{n=1}^{+\infty}\sqrt{n+1}-\sqrt{n} \underset{\text{Telescoping.}}{=} \lim_{n\to+\infty}\sqrt{n}-\sqrt{1}$ 2

$= +\infty$. Diverges 2

Therefore $\sum(-1)^n\cdot\left(\sqrt{n+1}-\sqrt{n}\right)$ converges conditionally

2. [**13 Points**] Find the sum of the following series.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{+\infty} \ln(n) - \ln(n+1)$$ 3 Telescoping

$$= \ln(1) - \lim_{n \to +\infty} \ln(n) = -\infty$$ 3 1 . Diverges

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{2^n} = \sum_{n=0}^{+\infty} \frac{(-1)^n}{2^n} = \sum_{n=0}^{+\infty} \left(\frac{-1}{2}\right)^n$$ 2 2 Geometric Series

$$= \sum_{n=1}^{+\infty} \left(-\frac{1}{2}\right)^{n-1} = \frac{1}{1 - \frac{-1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$ 1 1

3. **[15 Points]** Determine if each of the given series converges or diverges. Justify your answer.

$$\sum_{n=1}^{\infty}\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^{n}$$

$$\lim_{n\to+\infty}\sqrt[n]{\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^{n}} \;1\; = \lim_{n\to+\infty}\frac{\sqrt{2n}}{3\sqrt{n}+4} \;1$$

$$= \lim_{n\to+\infty}\frac{\sqrt{2}\cdot\cancel{\sqrt{n}}}{3\cdot\cancel{\sqrt{n}}} = \frac{\sqrt{2}}{3} \;1$$

$\rho = \frac{\sqrt{2}}{3} < 1$ 1 1 By root test 1, the series Converges. 1

$$\sum_{n=1}^{\infty}\frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})} = \sum_{n=1}^{+\infty}\frac{(\ln(n))^2}{8n^{3/2}+9n}$$

$$\frac{\ln(n)^2}{8n^{3/2}+9n} \leq \frac{\ln(n)^2}{8\cdot n^{3/2}} = \frac{(\ln(n))^2}{8\cdot n^{1.5}} \leq \frac{(n^{0.1})^2}{8\cdot n^{1.5}}$$

for n large enough 4

$$= \frac{n^{0.2}}{8\cdot n^{1.5}} = \frac{1}{8n^{1.3}} \leq \frac{1}{n^{1.3}}$$

$\sum \frac{1}{n^{1.3}}$ Converges (p-series, $p=1.3>1$). 2

By D.C.T, 1 $\sum \frac{(\ln(n))^2}{\sqrt{n}\,[8n+9\sqrt{n}]}$ Converges. 1

4. [20 Points] Determine whether the given series converges conditionally, absolutely, or diverges.

$$\sum_{n=1}^{\infty} \frac{\sin(n!)}{1+n^2}$$

$$\left| \frac{\sin(n!)}{1+n^2} \right| \leq \frac{1}{1+n^2} \leq \frac{1}{n^2} \quad 2$$

$\sum \frac{1}{n^2}$ Converges (p-series, $p = 2 > 1$). 3

By D.C.T, $\sum \left| \frac{\sin(n!)}{1+n^2} \right|$ Converges 3

$\Rightarrow \sum \frac{\sin(n!)}{1+n^2}$ 2 Converges absolutely.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n (n!)^2}{(2n)!}$$

$$\frac{\left| (-1)^{n+1} \cdot 3^{n+1} \cdot (n+1)!^2 / (2n+2)! \right|}{\left| (-1)^n \cdot 3^n \cdot (n!)^2 / (2n)! \right|} \; 2 \; = \frac{3 \cdot (n+1)^2}{(2n+2)(2n+1)} \quad 2$$

$$= \frac{3n^2 + 6n + 3}{4n^2 + 6n + 2} \xrightarrow[n \to +\infty]{} \frac{3}{4} = \rho \quad 4$$

1 $\rho < 1 \Rightarrow \sum |\cdots\cdots|$ 1 converges by ratio Test 1

$\Rightarrow \sum (-1)^n \cdot \frac{3^n (n!)^2}{(2n)!}$ Converges absolutely. 2

5. **[15 Points]** Find the radius and interval of convergence of the following series.

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{\sqrt{n}}$$

$$\left|\frac{(-1)^{n+1}\cdot(x+2)^{n+1}/\sqrt{n+1}}{(-1)^n\cdot(x+2)^n/\sqrt{n}}\right| = \frac{\sqrt{n}}{\sqrt{n+1}}\cdot\frac{|x+2|^{n+1}}{|x+2|^n}$$

$$= \frac{\sqrt{n}}{\sqrt{n+1}}\cdot|x+2| \xrightarrow[n\to+\infty]{} |x+2| = \rho \quad 5$$

- $\rho<1 \iff |x+2|<1 \iff -1<x+2<1 \iff -3<x<-1$
series converges absolutely $\forall x\in\,]-3,-1[$. 4

- $\rho=1 \iff x=-3$ or $x=-1$.

$x=-3$: $\sum(-1)^n\frac{(-1)^n}{\sqrt{n}} = \sum\frac{1}{\sqrt{n}} = \sum\frac{1}{n^{1/2}}$ 2 Diverges ($p=\frac{1}{2}>1$) p-series

$x=-1$: $\sum(-1)^n\cdot\frac{1}{\sqrt{n}}$ Converges ($\frac{1}{\sqrt{n}}\downarrow$, $\frac{1}{\sqrt{n}}\to0$, Alt. series By Leibniz test. 2

$\Rightarrow$ Interval of convergence is $]-3,-1]$ 1

Radius $=\frac{(-1)-(-3)}{2}=1$ 1

6. **[20 Points]**

(a) How many terms do we need to estimate the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}$ with an error less than 0.0001? Justify your answer.

$|L-S_n|<u_{n+1}$ 1

$u_{n+1}<0.0001$

$\frac{1}{2(n+1)+1}<0.0001$ 1

$2n+3>10000$

$2n>9997$ 1

$n>\frac{9997}{2}=4998.5$ 1

choose $n=4999$

we need 4999 terms

conditions:

$\frac{1}{2n+1}>0$

$\frac{1}{2n+1}\downarrow$ 1

$\frac{1}{2n+1}\to0$

The series is alt.

$$|r| < 1$$
$$|-x| < 1$$
$$|x| < 1 \qquad -1 < x < 1$$

(b) For what values of x does the series $\sum_{n=0}^{\infty}(-1)^n x^n$ converge? Find its sum.

$$\sum_{n=0}^{+\infty} (-1)^n \cdot x^n = \sum_{n=1}^{+\infty} (-1)^{n-1} \cdot x^{n-1} = \sum_{n=1}^{+\infty} (-x)^{n-1} \quad 1$$

$$= \frac{1}{1-(-x)} = \frac{1}{1+x} \quad 2$$

Geometric series

$|-x| = |x| < 1$ 2

converges for $x \in]-1,1[$.

(c) Deduce that $\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$ for very x in the interval $(-1,1)$.

$$\sum_{n=0}^{+\infty} (-1)^n \cdot x^{2n} = \sum_{n=0}^{+\infty} (-1)^n \cdot (x^2)^n = \frac{1}{1+x^2} \quad 3$$

with $|x^2| < 1 \Leftrightarrow |x|^2 < 1 \Leftrightarrow |x| < 1$

$\Leftrightarrow x \in]-1,1[$. 2

(d) Deduce that $\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for very x in the interval $(-1,1)$.

$$\frac{1}{1+x^2} = \sum_{n=0}^{+\infty} (-1)^n \cdot x^{2n} \quad \forall x \in]-1,1[.$$

$$\Rightarrow \tan^{-1}(x) = \int_0^x \frac{1}{1+t^2} \cdot dt = \sum_{n=0}^{+\infty} \int_0^x (-1)^n \cdot t^{2n} \, dx \quad 5$$

$$= \sum_{n=0} (-1)^n \cdot \frac{x^{2n+1}}{2n+1}$$

7. [**5 Points**] Find a so that the series

$$\sum_{n+1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$$

converges. Justify your answer.

$$\sum_{n=1}^{+\infty} \frac{a}{n+2} - \frac{1}{n+4} = \sum_{n=1}^{+\infty} \frac{a(n+4)-(n+2)}{(n+2)\cdot(n+4)}$$

$$= \sum_{n=1}^{+\infty} \frac{(a-1)\cdot n + (4a-2)}{n^2+6n+8} \quad 1 \quad = S$$

If $a=1$! $S = \sum_{n=1}^{+\infty} \frac{2}{n^2+6n+8}$ Converges 2

$\left(\frac{2}{n^2+6n+8} \leq \frac{2}{n^2}, \sum \frac{1}{n^2} \text{ Converge } (p\text{-series}, p=2>1) \right.$

$\left. \Rightarrow \text{By D.C.T}, \sum \frac{2}{n^2+6n+8} \text{ Converge} \right)$

If $a-1>0$: $\lim_{n\to+\infty} \frac{(a-1).\ n+(4a-2)/n^2+6n+8}{\frac{1}{n}}$ 1 $=(a-1)$

$0<a-1<L.$

$\sum \frac{1}{n}$ Diverge ($p=1$, pseries)

By L.C.T: $\sum \frac{a}{n+2} - \frac{1}{n+4}$ Diverge

If $a-1<0$: $\lim_{n\to+\infty} \frac{(1-a).\ n-(4a-2)/n^2+6n+8}{\frac{1}{n}}$ 1 $= 1-a$

$1-a>0$, $\sum \frac{1}{n}$ dirg $\Rightarrow \sum \frac{(1-a)\cdot n-(4a-2)}{}$ ~~Converge~~ Dirrg

$\Rightarrow S$ dirrge

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Max

Student's Name: Saadallah Solh

Student's ID: 201408954

$\frac{97}{100}$

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(Keep it empty)

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Total	/100

1. **[12 Points]** Show that the series $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$ converges conditionally.

$$\sum_{n=1}^{\infty}\left|(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)\right| = \sum_{n=1}^{+\infty}\left|\sqrt{n+1}-\sqrt{n}\right| = \sum_{n=1}^{+\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

$$= \sum_{n=1}^{\infty}\left[\left(\sqrt{n+1}-\sqrt{n}\right)\frac{\left(\sqrt{n+1}+\sqrt{n}\right)}{\left(\sqrt{n+1}+\sqrt{n}\right)}\right] = \sum_{n=1}^{\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n+1}+\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{2} > 0.$$

6

then $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$ and $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$ has same nature of convergenc by LCT

and $\sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$ diverges (P-series $p=\frac{1}{2}<1$)

then $\sum\left|(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)\right|$ diverges.

and $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$

for $n>1$; $a_n = \left(\sqrt{n+1}-\sqrt{n}\right)$ ~~is continuous~~ $n+1>n$; $\sqrt{n+1}>\sqrt{n}$; $\sqrt{n+1}-\sqrt{n}>0 \Rightarrow a_n>0$

Let $f(x)=\sqrt{n+1}-\sqrt{n}$; $f'(x)=\frac{1}{2\sqrt{n+1}}-\frac{1}{2\sqrt{n}}<0$,

then $a_n=\sqrt{n+1}-\sqrt{n}$ is decreasing.

6

and $\lim_{n\to\infty}u_n = \lim_{n\to\infty}\sqrt{n+1}-\sqrt{n} = \lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}} = 0$

then by alternating series test ~~the~~ series

$\sum(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$ converges. ~~absolutely~~.
so converges conditionally.

2. **[13 Points]** Find the sum of the following series.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty}\left(\ln(n) - \ln(n+1)\right)$$

Let $a_n = \ln(n)$; $a_{n+1} = \ln(n+1)$

$$\Rightarrow \sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty}(a_n - a_{n+1}) = a_1 - \lim_{n\to\infty} a_n$$

$$= 0 - \lim_{n\to\infty} \ln(n) = -\infty$$

by telescoping series.

7

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{2^n}$$; $\cos(n\pi) = 1$ for n ~~odd~~ even

$\cos(n\pi) = -1$ for n ~~even~~ odd

then $\cos(n\pi) = (-1)^n$

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{2^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} = \sum_{n=0}^{\infty}\left(\frac{-1}{2}\right)^n.$$

this is a geometric series with $r = -\frac{1}{2}$

then $$\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{2^n} = \sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n = \cancel{\sum_{n=1}^{\infty}\left(\frac{-1}{2}\right)^{n-1}}$$

6

$$= \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

3. **[15 Points]** Determine if each of the given series converges or diverges. Justify your answer.

$$\sum_{n=1}^{\infty}\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n \; ;$$

$$\sqrt[n]{a_n} = \sqrt[n]{\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n} = \frac{\sqrt{2n}}{3\sqrt{n}+4} = \frac{\sqrt{2}\cdot\sqrt{n}}{3\sqrt{n}+4} \xrightarrow[n\to+\infty]{} \frac{\sqrt{2}}{3} = 0.47 = \rho$$

$\rho < 1$

so by root test $\sum\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n$ converges.

7

$$\sum_{n=1}^{\infty}\frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})} = \sum_{n=1}^{\infty}\frac{(\ln n)^2}{8n^{3/2}+9n}$$

$$\lim_{n\to\infty}\frac{\frac{(\ln n)^2}{8n^{3/2}+9n}}{\frac{(\ln(n))^2}{n^{3/2}}} = \lim_{n\to\infty}\frac{(\ln n)^2}{8n^{3/2}\left(1+\frac{9}{8}\frac{1}{n^{1/2}}\right)} \times \frac{n^{3/2}}{(\ln n)^2} = \frac{1}{8} > 0$$

8

then $\sum\frac{(\ln n)^2}{8n^{3/2}+9n}$ and $\sum\frac{(\ln n)^2}{n^{3/2}}$ has same nature by LCT

and $\ln n < n^{0.1}$

$(\ln n)^2 < n^{0.2}$

$\frac{(\ln n)^2}{n^{3/2}} < \left(\frac{n^{0.2}}{n^{3/2}} = \frac{1}{n^{1.3}}\right)$; $\sum_{n=1}^{\infty}\frac{1}{n^{1.3}}$ converges (p-series $p=1.3>1$)

then by DCT, $\sum\frac{(\ln n)^2}{n^{3/2}}$ converges.

$$\Rightarrow \sum_{n=1}^{\infty}\frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})} \text{ converges.}$$

4. [**20 Points**] Determine whether the given series converges conditionally, absolutely, or diverges.

$$\sum_{n=1}^{\infty} \frac{\sin(n!)}{1+n^2}$$

let. $\sum_{n=1}^{\infty} \left|\frac{\sin(n!)}{1+n^2}\right|$; $0 \leq |\sin(n!)| \leq 1$

$\left|\frac{\sin(n!)}{1+n^2}\right| \leq \frac{1}{1+n^2} \leq \frac{1}{n^2}$

10

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series , $p=2>1$)

then by DCT $\sum \left|\frac{\sin(n!)}{1+n^2}\right|$ converges.

so the series converges absolutely

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n (n!)^2}{(2n)!}$$

$\sum_{n=1}^{\infty} \left|(-1)^n \frac{3^n (n!)^2}{(2n)!}\right| = \sum_{n=1}^{\infty} \left|\frac{3^n (n!)^2}{(2n)!}\right|$

let $a_n = \frac{3^n (n!)^2}{(2n)!}$

$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{3^{n+1}((n+1)!)^2}{(2n+2)!} \times \frac{(2n)!}{3^n (n!)^2} \right| = \left| \frac{3(n+1)^2 (n!)^2 \cdot (2n)!}{(2n+2)! \, (n!)^2} \right|$

10

$= \left| \frac{3(n+1)^2}{(2n+2)(2n+1)} \right| = \left| \frac{3(n+1)}{2(2n+1)} \right| \xrightarrow[n \to \infty]{} \frac{3}{4} < 1$

then by ratio test $\sum \left|(-1)^n \frac{3^n (n!)^2}{(2n)!}\right|$ converges

So the series converges absolutly

5. **[15 Points]** Find the radius and interval of convergence of the following series.

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{\sqrt{n}}$$

$$\sum_{n=1}^{\infty}\left|(-1)^n\frac{(x+2)^n}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{|x+2|^n}{\sqrt{n}} \; ; \quad \frac{|x+2|^{n+1}}{\sqrt{n+1}} \times \frac{\sqrt{n}}{|x+2|^n}$$

$$= \frac{|x+2|\sqrt{n}}{\sqrt{n+1}} \xrightarrow[n\to\infty]{} |x+2| = \rho$$

5

the series converges absolutly if $\rho < 1 \Rightarrow |x+2| < 1$

$-1 < x+2 < 1 \; ; \; -3 < x < -1$

4

2 for $x = -3$; $\sum_{n=1}^{\infty}\frac{(-1)^n \cdot (-1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ diverges (p series $p = \frac{1}{2} < 1$)

for $x = -1$; $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$; $\cancel{\sum \left|\frac{(-1)^n}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}}$ ~~diverges~~

let $a_n = \frac{1}{\sqrt{n}}$; $a_n > 0$ and $a_n \xrightarrow{n\to\infty} 0$; $a_n \downarrow$

2 then by alternate series test $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ converges

1 so the interval of convergence is $]-3 ; -1]$

the radius of convergence is ~~X~~. $\frac{(-1)-(-3)}{2} = \frac{-1+3}{2} = \frac{2}{2} = 1$

Radius $= 1$

6. **[20 Points]**

(a) How many terms do we need to estimate the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}$ with an error less than 0.0001? Justify your answer.

$$S_n = \sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1} = \sum_{n=1}^{\infty}(-1)^n \cdot a_n \quad \text{where } a_n = \frac{1}{2n+1}.$$

~~let~~ $|S_n - L| < a_{n+1} < 0.0001$

$\frac{1}{2n+2+1} < 0.0001$

$\frac{1}{2n+3} < 0.0001 \; ; \; 2n+3 > 10000 \; ;$

$n > 4998.5 \Rightarrow n \geq 4999$

$n = 4999$

4

could ... ?

(b) For what values of x does the series $\sum_{n=0}^{\infty}(-1)^n x^n$ converge? Find its sum.

$\sum_{n=0}^{\infty}(-1)^n x^n = \sum_{n=0}^{\infty}(-x)^n$. it is a geometric series, it converges if $|-x|<1$

$\Rightarrow -1 < x < 1$

for $x \in]-1,1[$ $\sum_{n=0}^{\infty}(-x)^n = \cancel{\frac{1}{}} \sum_{n=1}^{\infty}(-x)^{n-1} = \frac{1}{1+x}$. 5

(c) Deduce that $\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$ for very x in the interval $(-1,1)$.

Let $f(x) = \frac{1}{1+x} = \sum(-1)^n x^n$ for $-1 < x < 1$

then $f(x^2) = \frac{1}{1+x^2} = \sum(-1)^n (x^2)^n = \sum(-1)^n x^{2n}$ for $-1 < x^2 < 1$

5

$\Rightarrow -1 < x < 1$

(d) Deduce that $\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for very x in the interval $(-1,1)$.

$\int f(x^2) = \int \left(\sum_{n=0}^{\infty}(-1)^n x^{2n}\right) dx$. for $-1 < x < 1$

$\int \frac{1}{1+x^2} dx = \sum_{n=0}^{\infty}\left[(-1)^n \int x^{2n} dx\right]$ 5

$\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for $-1 < x < 1$

7. [**5 Points**] Find a so that the series

$$\sum_{n=1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$$

converges. Justify your answer.

$$\sum_{n=1}^{\infty}\left(\frac{\alpha}{n+2}-\frac{1}{n+4}\right)=\sum_{n=1}^{\infty}\left(\frac{\alpha}{n+2}-\frac{1}{n+3}+\frac{1}{n+3}-\frac{1}{n+4}\right)$$

$$=\sum_{n=1}^{\infty}\left(\frac{\alpha}{n+2}-\frac{1}{n+3}\right)+\sum_{n=1}^{\infty}\left(\frac{1}{n+3}-\frac{1}{n+4}\right)$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n+3}-\frac{1}{n+4}\right)=\frac{1}{4}-\lim_{n\to\infty}\frac{1}{n+3}=\frac{1}{4} \text{ converges}$$

by telscoping serie.

so $\sum_{n=1}^{\infty}\left(\frac{\alpha}{n+2}-\frac{1}{n+4}\right)$ converges if $\sum_{n=1}^{\infty}\left(\frac{\alpha}{n+2}-\frac{1}{n+3}\right)$ converges.

for $\alpha=1$; $\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+3}\right)=\frac{1}{3}-\lim_{n\to\infty}\frac{1}{n+2}=\frac{1}{3}$ convergs

for $\alpha\neq 1$, let $b=\alpha+(-1)$; $\alpha=b+1$; ~~& b=~~ & $b\neq 0$

$$\sum_{n=1}^{\infty}\left(\frac{\alpha}{n+2}-\frac{1}{n+3}\right)=\sum_{n=1}^{\infty}\left(\frac{b+1}{n+2}-\frac{1}{n+3}\right)=\sum_{n=1}^{\infty}\left(\frac{b}{n+2}+\frac{1}{n+2}-\frac{1}{n+3}\right)$$

$\sum_{n=1}^{\infty}\frac{b}{n+2}$ ~~$\lim \frac{b}{n+2}$ and $\frac{b}{n+2}$ $\frac{1}{n+2}$~~

and $\lim_{n\to\infty}\frac{\frac{1}{n+2}}{\frac{1}{n}}=1$, and $\sum\frac{1}{n}$ div. (p seris $p=1\leq 1$)

$\Rightarrow$ by LCT $\sum_{n=1}^{\infty}\frac{1}{n+2}$ diverges

$\Rightarrow \sum_{n=1}^{\infty}\frac{b}{n+2}$ diverges by DCT then $\sum_{n=1}^{\infty}\left(\frac{b}{n+2}+\frac{1}{n+2}-\frac{1}{n+3}\right)$ diverges

f sol $\sum\left(\frac{\alpha}{n+2}-\frac{1}{n+4}\right)$ donveges for $\alpha=1$.

5

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Exam II, Spring 2015

Duration: 60 minutes

INSTRUCTIONS: This exam consists of 8 pages and 7 problems. Check that none is missing. Answer the questions in the space provided for each problem; if more space is needed, you may use the back pages. To receive full credits, you have to justify your answers.

Student's Name: Hani Sami

Student's ID: 201405724

Average

$\frac{74}{100}$

Grading scheme
(Keep it empty)

Question 1	/12
Question 2	/13
Question 3	/15
Question 4	/20
Question 5	/15
Question 6	/20
Question 7	/5
Total	/100

1. **[12 Points]** Show that the series $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$ converges conditionally.

$$\sqrt{n+1}-\sqrt{n} = \sqrt{n+1}-\sqrt{n} \times \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$$= \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}} \longrightarrow 0$$

$$\sum_{n=1}^{+\infty}(-1)^n\sqrt{n+1}-\sqrt{n} = \sum_{n=1}^{+\infty}(-1)^n\frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to+\infty}\left|\frac{U_{n+1}}{U_n}\right| = \lim_{n\to+\infty}\left|\frac{\sqrt{(n+1)+1}+\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n}}\right| = \lim_{n\to+\infty}\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to+\infty}\frac{\frac{1}{2\sqrt{n+2}}+\frac{1}{2\sqrt{n+1}}}{\frac{1}{2\sqrt{n+1}}+\frac{1}{2\sqrt{n}}}$$

$$= 0 < 1$$ converges conditionally.

2. [**13 Points**] Find the sum of the following series.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \ln n - \ln(n+1)$$

telescoping series $= \cancel{\ln 1 + \lim \ln n}$

$\ln 1 - \ln 2$

$\ln 2 - \ln 3$

$\ln 3 - \ln 4$

$\ln(n-1) - \ln n$

$\ln n - \ln(n+1)$

$\sum_{n=1}^{+\infty} \ln\left(\frac{n}{n+1}\right) = \ln 1 - \ln(n+1)$

$= -\ln(n+1)$

$= -\infty$

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{2^n} = \frac{(-1)^n}{2^n} \quad n \geq 0.$$

$\frac{\cos \pi}{2}$

$= \left(\frac{-1}{2}\right)^n$ ~~geometric series~~ $\cos(n\pi) \leq 1.$

geometric series

$S_n = \frac{1}{1+\frac{1}{2}}$

$= \frac{1}{\frac{3}{2}}$

$= \frac{2}{3}$

$\cos(n\pi)$

$\frac{1}{2^n}$ $\frac{1}{2^n}$. DCT

$\sum \frac{1}{2^n}$ converges geometric series.

$\cancel{\frac{\cos n\pi}{2^n}}$

$\sum \frac{1}{2^n} = \frac{1}{1-\frac{1}{2}} = 2.$ $\sum \frac{\cos(n\pi)}{2^n}$

3. **[15 Points]** Determine if each of the given series converges or diverges. Justify your answer.

$$\sum_{n=1}^{\infty} \left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n$$

apply root test $\lim_{n\to+\infty} \sqrt[n]{\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n} = \lim_{n\to+\infty} \frac{\sqrt{2n}}{3\sqrt{n}+4}$

$$= \frac{\sqrt{2}\times\sqrt{n}}{3\sqrt{n}} = \frac{\sqrt{2}}{3} < 1$$

7

converges by root test.

$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})} = \sum_{n=1}^{+\infty} \frac{(\ln n)^2}{8n\sqrt{n}+9n}$$

note that $8n\sqrt{n}+9n > 8n\sqrt{n}$

8

$$\frac{(\ln n)^2}{8n\sqrt{n}+9n} < \frac{(\ln n)^2}{8n\sqrt{n}}$$ for n large.

$(\ln n)^2 < n^{0,1}$ for n large.

$$\frac{(\ln n)^2}{8n\sqrt{n}} < \frac{n^{0,1}}{8n\sqrt{n}} = \frac{1}{8n^{1,4}}$$ (P series $P = 1,4 > 1$ converges) $\sum \frac{1}{n^{1,4}}$

$\rightarrow \frac{(\ln n)^2}{8n\sqrt{n}}$ converge D.C.T ~~and~~ $\rightarrow$ $\sum \frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})}$ converge D.C.T

4. **[20 Points]** Determine whether the given series converges conditionally, absolutely, or diverges.

$$\sum_{n=1}^{\infty} \frac{\sin(n!)}{1+n^2}$$

$n > 1 \rightarrow \sin(n)^{?} < 1 \rightarrow \sin(n!) < 1$

$\cancel{\sin(n!) <}$

$\left|\frac{\sin(n!)}{1+n^2}\right| \cancel{<} \cancel{\frac{1}{1+n^2}}$ and $\left|\frac{\sin(n!)}{1+n^2}\right| < \left|\frac{1}{1+n^2}\right|$

2

$\left|\frac{1}{1+n^2}\right| < \left|\frac{1}{n^2}\right|$ $\sum \frac{1}{n^2}$ (p series $\cancel{p>1}$. $p=2>1$ converges) $\rightarrow \sum \left|\frac{1}{1+n^2}\right|$ converges by D.C.T.

2 1

$\rightarrow \sum \frac{\sin(n!)}{1+n^2}$ converges ~~conditionally~~ absolutely D.C.T

way of writing is wrong

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n (n!)^2}{(2n)!}$$

$\left|(-1)^n \frac{3^n (n!)^2}{(2n)!}\right| = \frac{3^n \times (n!)^2}{2n!}$

ratio test: $\lim_{n\to+\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{n+1} \times (n+1)! \times (n+1)!}{(2n+2)!} \times \frac{(2n)!}{3^n\, n! \times n!}$

10

$= \lim_{n\to+\infty} \frac{3 \times (n+1) \times (n+1)}{(2n+1) \times (2n+2)} = \cancel{\frac{3n^2}{4n^2}} \lim_{n\to+\infty} \frac{3n^2}{4n^2}$

$= \frac{3}{4} < 1$

$\sum_{n=1}^{+\infty} (-1)^n \frac{3^n (n!)^2}{(2n)!}$ converges absolutely by ~~ra~~ root test.

5. **[15 Points]** Find the radius and interval of convergence of the following series.

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{\sqrt{n}}$$

$$\lim_{n\to+\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to+\infty}\frac{|x+2|^{n+1}}{\sqrt{n+1}} \times \frac{\sqrt{n}}{|x+2|^n} = |x+2|$$

$$|x+2| < 1$$

$$-1 < x+2 < 1 \to -3 < x < -1 \text{ converges}$$

$$\cancel{1 < x < 3}$$

14

On $x = -3$: $\sum_{n=1}^{+\infty}(-1)^n\frac{(-1)^n}{\sqrt{n}} = \sum_{n=1}^{+\infty}\frac{1}{\sqrt{n}}$ diverges (P series $P = 0{,}5 < 1$).

On $x = -1$: $\sum_{n=1}^{+\infty}(-1)^n \times \frac{(1)^n}{\sqrt{n}} = \sum_{n=1}^{+\infty}\frac{(-1)^n}{\sqrt{n}}$ by alternating test $\frac{1}{\sqrt{n}} > 0$

$$\lim_{n\to+\infty}\frac{1}{\sqrt{n}} = 0$$

$$\left(\frac{1}{\sqrt{n}}\right)' = -\frac{1}{2\sqrt{n}} < 0$$

$\sum_{n=1}^{+\infty}(-1)^n\boxed{(1)}^n$

$\sum_{n=1}^{+\infty}\frac{(-1)^n}{\sqrt{n}}$ converges

interval of convergence

$$-3 < x \leq -1$$

$$-1 < x < -3$$

$$-3 < x \leq -1$$

radius $= \frac{-1+3}{2} = 1$

6. **[20 Points]**

(a) How many terms do we need to estimate the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}$ with an error less than 0.0001? Justify your answer.

Conditions

$$\cancel{\left|\frac{1}{u_{n+1}}\right|} < 10^{-4}$$

$$|E| = |u_{n+1}| < 10^{-4}$$

$$\cancel{(2n+1) < 10^{-4}}$$

4

$$|\text{error}|: \frac{1}{2(n+1)+1} < \frac{1}{10^4}$$

$$2n+3 > 10^4$$

$$2n > 10^4 - 3 \to n > \frac{10^4-3}{2}$$

$n > 4998{,}5$ we need $n = 499$.

$x < 0$

$x > 0$

$(-1)^n x^n$

$0 < x < 1$

$\frac{1}{1-x}$

(b) For what values of x does the series $\sum_{n=0}^{\infty}(-1)^n x^n$ converge? Find its sum.

$|(-1)^n x^n| = x^n \to \sum_{n=0}^{+\infty} x^n = \sum_{n=1}^{+\infty} x^{n-1}$ geometric series,

$-1 < x < 1$. Sum $= \frac{1}{1-(-x)} = \frac{1}{1+(x)} = \frac{1}{1+x}$.

2

(c) Deduce that $\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$ for very x in the interval $(-1,1)$.

~~for~~ replace x by x^2 when $-1 < x < 1$.

$$\sum_{n=0}^{+\infty} (-1)^n x^n = \frac{1}{1+(x^2)^n} \quad \frac{1}{1+x^{2n}}$$

(d) Deduce that $\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for very x in the interval $(-1,1)$.

$\int \left(\sum (-1)^n x^{2n}\right)$, $f(x) = x^{2n}$

$= \sum (-1)^n \int f(x) = \sum (-1)^n \frac{x^{2n+1}}{2n+1} = \int \left(\frac{1}{1+x^2}\right)$

5

$= \tan^{-1} x$.

page before

7. [**5 Points**] Find a so that the series

$$\sum_{n+1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$$

converges. Justify your answer.

$$\sum_{n+1}^{+\infty} \frac{an+a4-n-2}{(n+2)(n+4)}$$

$$= \sum_{n=1}^{+\infty} \frac{(a-1)n + 4a-2}{(n+2)(n+4)}.$$

if $(a-1)n = 0$

$$\rightarrow \sum_{n=1}^{+\infty} \frac{4a-2}{(n+2)(n+4)} = \frac{4a-2}{n^2+6n+8} < \frac{4a-2}{n^2}$$

P series $p=2$ converges

~~behave like~~

$$\lim_{n\to+\infty} \frac{4a-2}{n^2+6n+8}$$

$$\sum \frac{4a-2}{n^2+6n+8}$$ converge

if $(a-1)n \neq 0$

LEBANESE AMERICAN UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE AND MATHEMATICS

MTH 201 - CALCULUS III

Exam II, Spring 2015

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Min

Student's Name: Abbas Fayad

Student's ID: 201501699

$\boxed{\frac{31}{100}}$

Grading scheme
(Keep it empty)

Question 1	/12
Question 2	/13
Question 3	/15
Question 4	/20
Question 5	/15
Question 6	/20
Question 7	/5
Total	/100

1. **[12 Points]** Show that the series $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$ converges conditionally.

$a_n > b_n$

a_n b_n a_n div $\to$ b_n div

$a_n < b_n$

b_n a_n b_n conv $\to$ a_n conv

$$\cancel{\sum_{n=1}^{\infty}\left|(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)\right|}$$

$$\cancel{=\sum_{n=1}^{\infty}\sqrt{n+1}-\sqrt{n}}$$

$$\sum_{n=0}^{\infty}(-1)^{n+1}\left(\sqrt{n+1}-\sqrt{n}\right)$$

$$\sum_{n=0}^{\infty}\left|\sqrt{n+1}-\sqrt{n}\right| \quad \cancel{\text{Conv}} \ \text{Div}$$

Alt Series $\left(a_n > 0,\ a_n \downarrow,\ a \to 0\right)$

$n+1$ odd

$$\sum_{n=0}^{\infty} -\left(\sqrt{n+1}-\sqrt{n}\right)$$

$$\sum_{n=0}^{\infty}\sqrt{n}-\sqrt{n+1}$$

$$\sqrt{0}-\lim_{n\to\infty}\sqrt{n+1}$$

$\Rightarrow \infty$ diverge

$n+1$ even

$$\sum_{n=0}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$$

Conv (Alt Series $a_n > 0,\ a_n \downarrow,\ a_n \to 0$)

$\therefore \sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$ Converges Conditionally

2. **[13 Points]** Find the sum of the following series.

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$$

$\ln\left(\frac{1}{2}\right) + \ln\left(\frac{2}{3}\right) + \ln\left(\frac{3}{4}\right) + \ln\left(\frac{4}{5}\right) + \ln\left(\frac{5}{6}\right) +$

$$\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{2^n} = \sum \frac{(-1)^n}{2^n}$$

$\cos(n\pi) = (-1)^n$

$-1 < \cos(n\pi) < 1$

$\frac{-1}{2^n} < \frac{\cos(n\pi)}{2^n} < \frac{1}{2^n}$

$\left|\frac{\cos(n\pi)}{2n}\right| < \frac{1}{2^n} = \left(\frac{1}{2}\right)^n$

1

$\frac{1}{2^n} = \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = 2$ max

$-\left(\frac{1}{2}\right)^n = \frac{-1}{1-\frac{1}{2}} = -2$ min

> 0

3. **[15 Points]** Determine if each of the given series converges or diverges. Justify your answer.

$$\sum_{n=1}^{\infty}\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n$$

$$\sum_{n=1}^{\infty} \sqrt[n]{\left(\frac{\sqrt{2n}}{3\sqrt{n}+4}\right)^n}$$

$$\lim_{n\to\infty}\frac{\sqrt{2n}}{3\sqrt{n}+4} = \frac{\sqrt{2}n^{1/2}}{3n^{1/2}} = \cancel{1} \neq \rho \quad \text{~~Inconclusive~~} \quad \frac{\sqrt{2}}{3} \quad \text{~~Conv~~}$$

$$\rho = \frac{\sqrt{2}}{3} \quad \text{Conv} \left(\text{Root test } \rho = \frac{\sqrt{2}}{3} < 1\right)$$

7

$$\sum_{n=1}^{\infty}\underbrace{\frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})}}_{a_n} = \sum_{n=1}^{\infty}\frac{(\ln(n))^2}{n^{0.5}(8n+9n^{0.5})} = \sum_{n=1}^{\infty}\frac{(\ln(n))^2}{8n^{1.5}+9n}$$

n^{th} term test

$$\frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})} \simeq \underbrace{\frac{(\ln(n))^2}{8n^{1.5}}}_{b_n}$$

$$\frac{\frac{(\ln n)^2}{\sqrt{n}(8n+9\sqrt{n})}}{\frac{(\ln n)^2}{8n^{1.5}}} = 1$$

2

By LCT $\sum a_n$ & $\sum b_n$ have the same nature

$$\lim_{n\to\infty}\frac{(\ln n)^2}{8n^{1.5}} = \cancel{\frac{\infty}{\infty}} = 1 \quad \text{Conv}$$

$$\therefore \cancel{\sum\frac{(\ln(n))^2}{\sqrt{n}(8n+9\sqrt{n})}}$$

4. [**20 Points**] Determine whether the given series converges conditionally, absolutely, or diverges.

$$\sum_{n=1}^{\infty} \frac{\sin(n!)}{1+n^2}$$

$-1 \le \sin(n!) \le 1$

$n^2 < n^3$

$n^2+1 < n^3+1$

$\frac{1}{n^2+1} > \frac{1}{n^3+1}$

$\frac{\sin(n!)}{1+n^2} < \cancel{\frac{1}{n^3+1}}$

$\lim_{n\to\infty} \frac{\frac{1}{n^3+1}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{n^3}{n^3+1} = 1$ $\quad \sum \frac{1}{n^3}$ & $\frac{1}{n^3+1}$ have same nature

$\sum \frac{1}{n^3}$ Conv ($p=3>1$ p-series) $\therefore$ By L.C.T $\sum \frac{1}{n^3+1}$ Converg...

$\Rightarrow \sum$

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n (n!)^2}{(2n)!}$$

$$\frac{a_{n+1}}{a_n} = \frac{\left|\cancel{(-1)^{n+1}} \cdot \frac{3^{n+1}((n+1)!)^2}{(2(n+1))!}\right|}{\left|\cancel{(-1)^n} \cdot \frac{3^n (n!)^2}{(2n)!}\right|} = \frac{3^{n+1}((n+1)!)^2 \cdot 2n!}{(2n+2)!\, 3^n (n!)^2} = \frac{3 \cdot (n+1)^2 \cdot \cancel{(n!)^2}}{(2n+2)(2n+1)\cancel{(n!)^2}}$$

$(2n+2)(2n+1)$

$$= \frac{3n^2+6n+3}{4n^2+4n+2} \longrightarrow \frac{3}{4} = \rho \quad \left(\text{Converg Absolutely } \rho = \frac{3}{4} < 1\right)$$

10

5. **[15 Points]** Find the radius and interval of convergence of the following series.

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x+2)^n}{\sqrt{n}}$$

$$\left|\frac{(x+2)^n}{\sqrt{n}}\right|$$

6. **[20 Points]**

(a) How many terms do we need to estimate the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+1}$ with an error less than 0.0001? Justify your answer.

$$-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\frac{1}{13}$$

$$-\frac{1}{4}+\frac{1}{8}-\frac{1}{8}+\frac{1}{16}-\frac{1}{16}+\frac{1}{16}-\frac{1}{16}$$

1 $|L-S_n| < 0.0001$

$|L-S_n| < 10^{-5}$

(b) For what values of x does the series $\sum_{n=0}^{\infty}(-1)^n x^n$ converge? Find its sum.

$|r| < 1$

$x = r$

$|r| = |x|$

$|x| < 1$

$\sum_{n=0}^{\infty} (-1)^n (x)^n$

geometric series

Sum = ??

2

(c) Deduce that $\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$ for very x in the interval $(-1,1)$.

$|r| < 1$

$|r| = |x^2|$

$\frac{1}{1-r} = \frac{1}{1-x^2}$

$\sum_{n=0}^{\infty} (-1)^n (x^2)^n = \sum (-x^2)^n = \frac{1}{1-(-x^2)} = \frac{1}{1+x^2}$

geometric series.

(d) Deduce that $\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$ for very x in the interval $(-1,1)$.

7. [**5 Points**] Find a so that the series

$$\sum_{n+1}^{\infty}\left(\frac{a}{n+2}-\frac{1}{n+4}\right)$$

converges. Justify your answer.

$$\frac{an+4a-n-2}{(n+2)(n+4)}$$

Divg

if $a-1 \neq 0$

$$\frac{n(a-1)\;-(n+2)}{(n+2)\,(n+4)} = \frac{n(a-1)}{n^2+6n+8} \overset{?}{=} \frac{a-1}{n}$$

$a-1$